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Observations of Resonance between Satellites in a High-Drag Environment and High-Order Tesseral Terms of the Geopotential Expansion

JULY 1966

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Prepared for COMMANDER SPACE SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
LOS ANGELES AIR FORCE STATION
Los Angeles, California

NOTICE

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OBSERVATIONS OF RESONANCE BETWEEN SATELLITES IN A HIGH-DRAG ENVIRONMENT AND HIGH-ORDER TESSERAL TERMS OF THE GEOPOTENTIAL EXPANSION

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July 1966

Prepared for

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Los Angeles, California

FOREWORD

This report is published by the Aerospace Corporation, El Segundo, California, under Air Force Contract No. AF 04(695)-669. The report was authored by Taylor P. Gabbard of the Electronics Division.

This report, which documents research carried out from September 1965 through April 1966, was submitted on 23 August 1966 to Colonel Lew Allen, Jr., Hq. SSD, for review and approval.

The author acknowledges with gratitude the use of the results of the work of Richard J. Farrar and Lem Wong on satellites 1963 42A and 1963 35A, respectively. The author also thanks Professor W. M. Kaula for starting estimates of coefficients, D. C. O'Brien for his art work and computational assistance, and the Aerospace Corp. Computation and Data Processing Center personnel who provided the larger mathematical tools that made this study possible.

Approved

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Group Director
Advanced Planning
Satellite Systems Division

Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

Lew Allen, Jr., Colonel, USAF
Headquarters Space Systems Division
Space Systems Division
Air Force Systems Command

ABSTRACT

Previous works by others have presented evidence of a satellite resonance phenomenon with geopotential tesseral terms of order $m = 13$ and $m = 14$. This report presents an analysis of the data of five satellites and provides evidence for this resonance phenomenon with terms of order $m = 16$, $m = 17$, and $m = 32$. By using two atmospheric density models in the analysis, it was concluded that even in a high-drag environment one observes an underlying satellite resonance phenomenon with geopotential forces arising from high-degree and high-order tesseral terms of the geopotential expansion. An accurate determination of the descriptive parameters, difficult with the present low-altitude satellite data and atmospheric density models available, must therefore be the result of a joint geopotential-atmosphere study, or await the direct use of drag accelerometer data.

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SECTION I

INTRODUCTION

In earlier works by Anderle (Reference 1) and Yionoulis (Reference 2), evidence was presented of a satellite resonance phenomenon with geopotential tesseral terms of order m = 13 and m = 14. In this report, evidence is presented for the existence of this satellite resonance phenomenon with geopotential terms of order m = 16, m = 17, and m = 32.

Geopotential tesseral terms of degree n and order m discussed here are characterized by J_{nm} and L_{nm}, and are defined by the geopotential expansion

$$U = \frac{\mu}{r}\left[1 - \sum_{n=2}^{\infty} J_n \left(\frac{a_e}{r}\right)^n P_n(\sin \phi)\right.$$

$$\left. + \sum_{n=2}^{\infty} \sum_{m=1}^{n} J_{nm} \left(\frac{a_e}{r}\right)^n P_{nm}(\sin \phi) \cos m(L - L_{nm})\right] \qquad (1)$$

where

μ	the product of the earth's mass and the constant of gravitation G
a_e	the earth's radius
P_n	Legendre polynomials of the first kind
P_{nm}	the associated Legendre polynomials of the first kind
r	the geocentric radius
ϕ	the latitude
L	Greenwich east longitude

Results presented here are derived primarily from an analysis of the tracking data of satellites 1962 BE 1, 1963 55A, and 1965 79A, the nominal orbital characteristics of which are presented in Table I. The raw data are range, azimuth, and elevation obtained by the tracking network of the Air Force Satellite Control Facility.

Table I also contains nominal orbital data for the considerably higher-perigee satellites 1963 35A and 1963 42A. All the results presented in this report for satellites 1963 35A and 1963 42A are the work, respectively, of Mr. Lem Wong and Mr. Richard J. Farrar, both of Aerospace Corporation.

Table I. Nominal Satellite Orbital Characteristics

Satellite	Perigee Height (n mi)	Eccentricity	Inclination (deg)	Nodal Period (min)
1962 BE 1	125	0.017	82.0	91 0
1963 55A	105	0.014	64.9	90.0
1965 79A	115	0.009	75.1	89.7
1963 35A	165	0.002	81.9	90.8
1963 42A	155	0.005	89.9	90.9

SECTION II

PROCEDURES

The first step was to produce, for each pass of raw data reported, a smoothed reduced data set that represents the more numerous raw data of the pass. This was done by making a local least-squares fit for each pass of data and computing three sets of range, azimuth, and elevation data from the resulting best-fit 6-vector of position and velocity at the local epoch. The remaining and principal portion of the study, now to be described, was accomplished by using TRACE, the Aerospace Corporation Orbit Determination Computer Program basically described in Reference 3.

Procedures using TRACE are primarily of two types and can be outlined by the four following steps:

a. Through long-arc fitting to the compacted data, a best-fit 7-vector of position and velocity at epoch, and the drag parameter C_DA/W are obtained (where C_D is the drag coefficient, A the frontal area, and W the weight of the satellite). For the near-earth satellites investigated, geopotential forces and atmospheric drag have been considered sufficient representations of the forces acting upon these satellites. The basic geopotential used throughout this study has been a 6th-degree and order model of Guier, which is unavailable in the open literature. Some experimentation with atmospheric density models was made by using the Lockheed-Jacchia (Reference 4) and Jacchia 1964 (Reference 5) models.

b. In order to determine how well the best-fit 7-vector, and the force model used in its determination, represent the motion of the satellites, the next step is to hold these quantities fixed and to solve for noncorrelated time biases for each pass of data. In the absence of any true clock errors, these time biases may be interpreted as errors in prediction of the time of arrival of the satellite at a station due to vector and force model errors, station location errors, etc. One may obtain in-track position error estimates by multiplying the time biases by minus inertial space velocity (i.e., speed) of the satellite. For satellites considered here, a nominal value for

this speed is 25 feet per millisecond. The minus (speed) results from the fact that a negative time bias signifies a predicted arrival at the station, which is earlier than what is actually observed, and this, in turn, signifies a positive in-track position prediction error.

c. If examination of the time bias patterns obtained in Step b reveals any gross deterministic force model errors, one returns to Step a and augments the state vector to include parameters that describe the candidate force model error selected.

d. In order to determine how well the candidate force model error removes the in-track error presented in the time bias pattern of Step b, one returns to Step b with the best-fit augmented state vector of Step c.

In Figures 1 through 7, are presented the time bias patterns resulting from the best-fit 7-vector for satellites 1962 BE 1, 1963 55A, 1965 79A, and 1963 42A; those for 1963 35A were not available in the present format. The force model used is indicated in each figure. In all cases, we observe periodic variations in the time bias patterns similar in nature to the variations in the along-track residual patterns of References 1 and 2.

The first 7-vector time bias patterns obtained were those employing the Lockheed-Jacchia atmospheric density model. However, since most of the satellites studied are in a relatively high-drag environment, we turned to the Jacchia 1964 model (the most sophisticated model locally available) to see if these variations might not be removed by a supposedly improved density model. The results of this atmospheric density model experiment may be seen by examining Figures 1 through 6.

The most important fact revealed is that even though the amplitude of the time bias patterns sometimes shows a strong dependence on the atmospheric density model used, the period of these variations is relatively insensitive to the choice of the density model.

It may still be argued that these variations in the time bias patterns are due to some remaining error in modeling the atmospheric density variations experienced by the satellites (or, more generally, to errors in modeling the drag acceleration); the principal effort of this study, however.

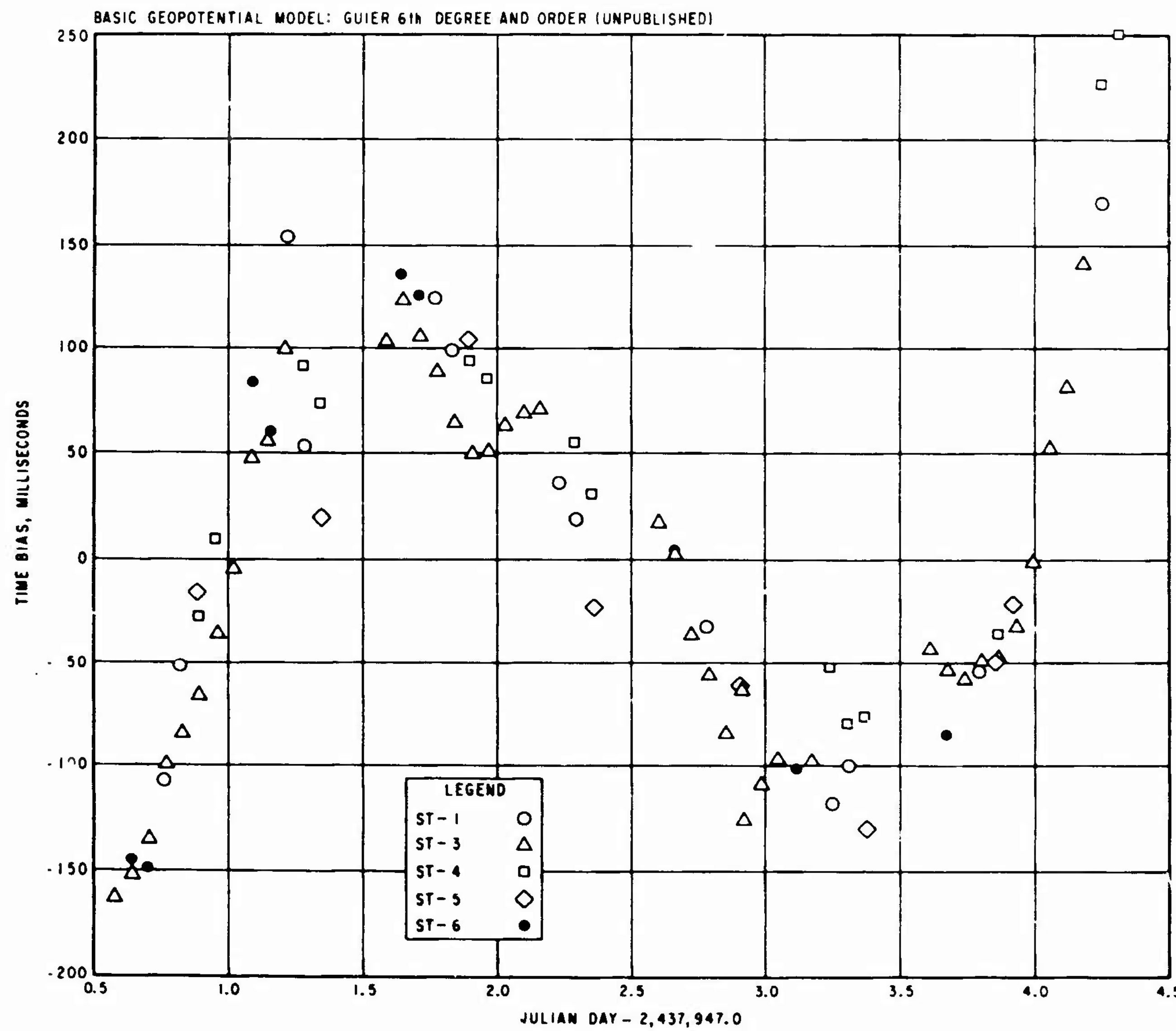

Figure 1. Time Bias Pattern without Resonant Terms: Satellite 1962 BE 1
(Lockheed-Jacchia Atmospheric Density Model)

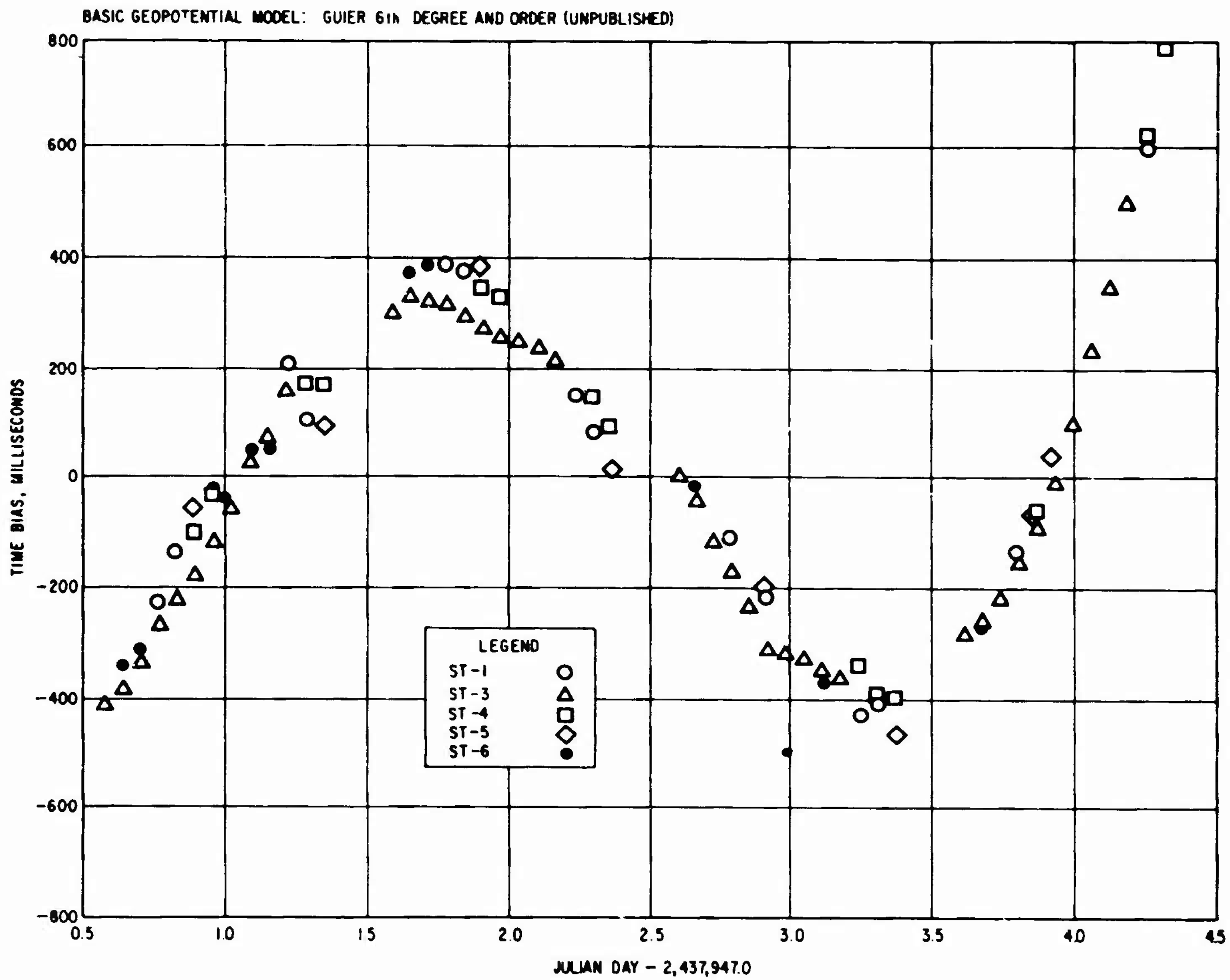

Figure 2. Time Bias Pattern without Resonant Terms: Satellite 1962 BE 1 (Jacchia 1964 Atmospheric Density Model)

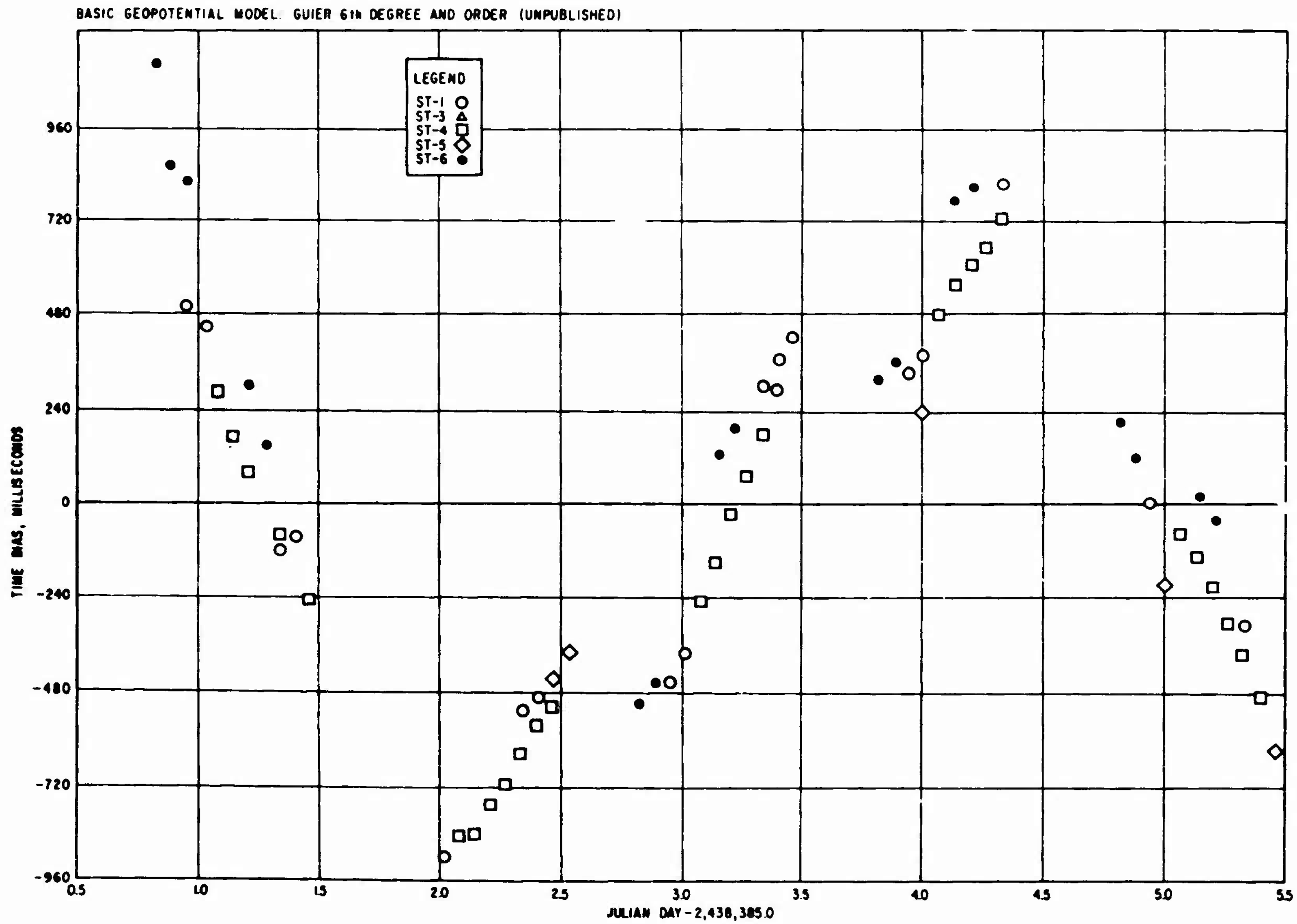

Figure 3. Time Bias Pattern without Resonant Terms: Satellite 1963 55A (Lockheed-Jacchia Atmospheric Density Model)

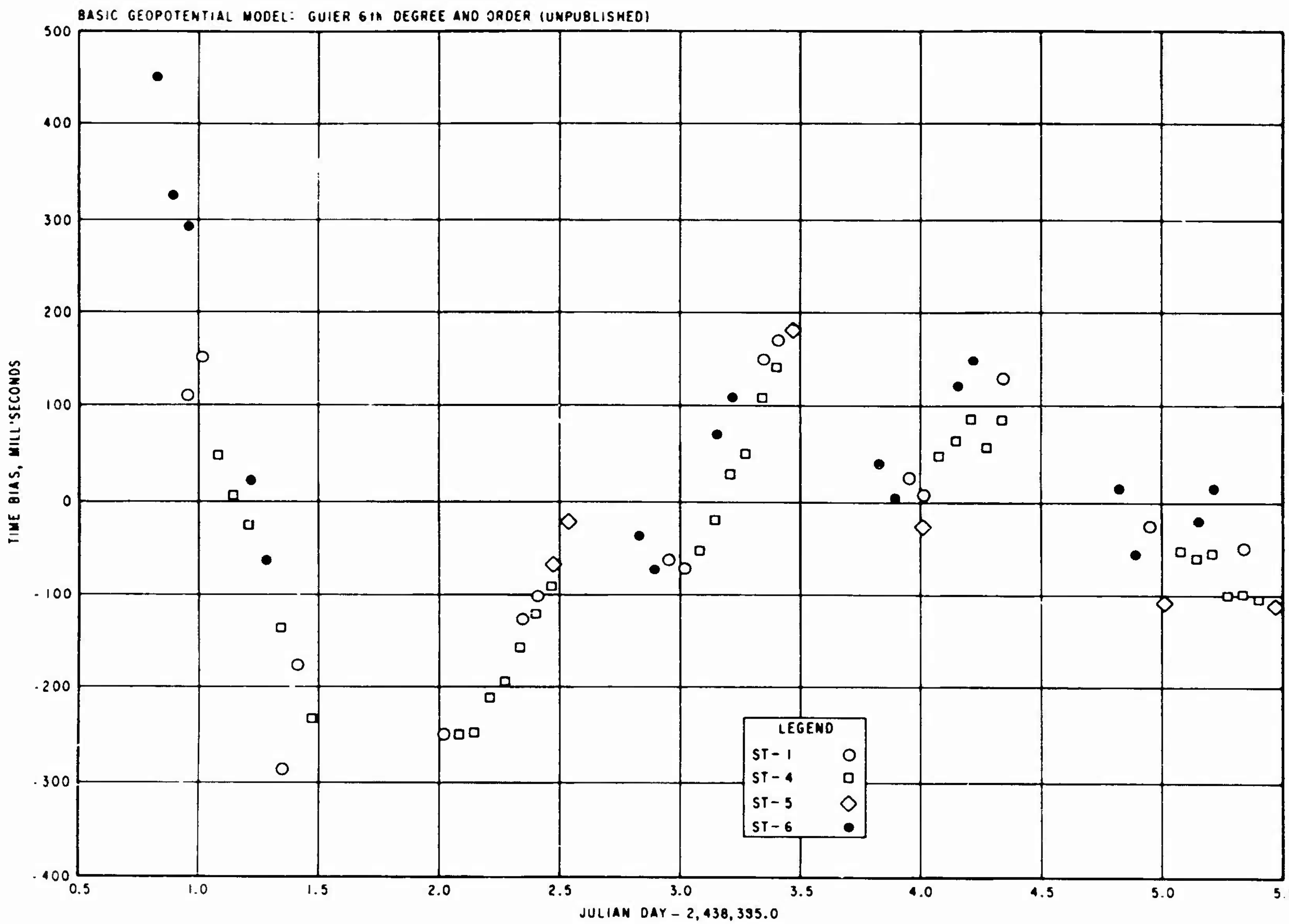

Figure 4. Time Bias Pattern without Resonant Terms: Satellite 1963 55A (Jacchia 1964 Atmospheric Density Model)

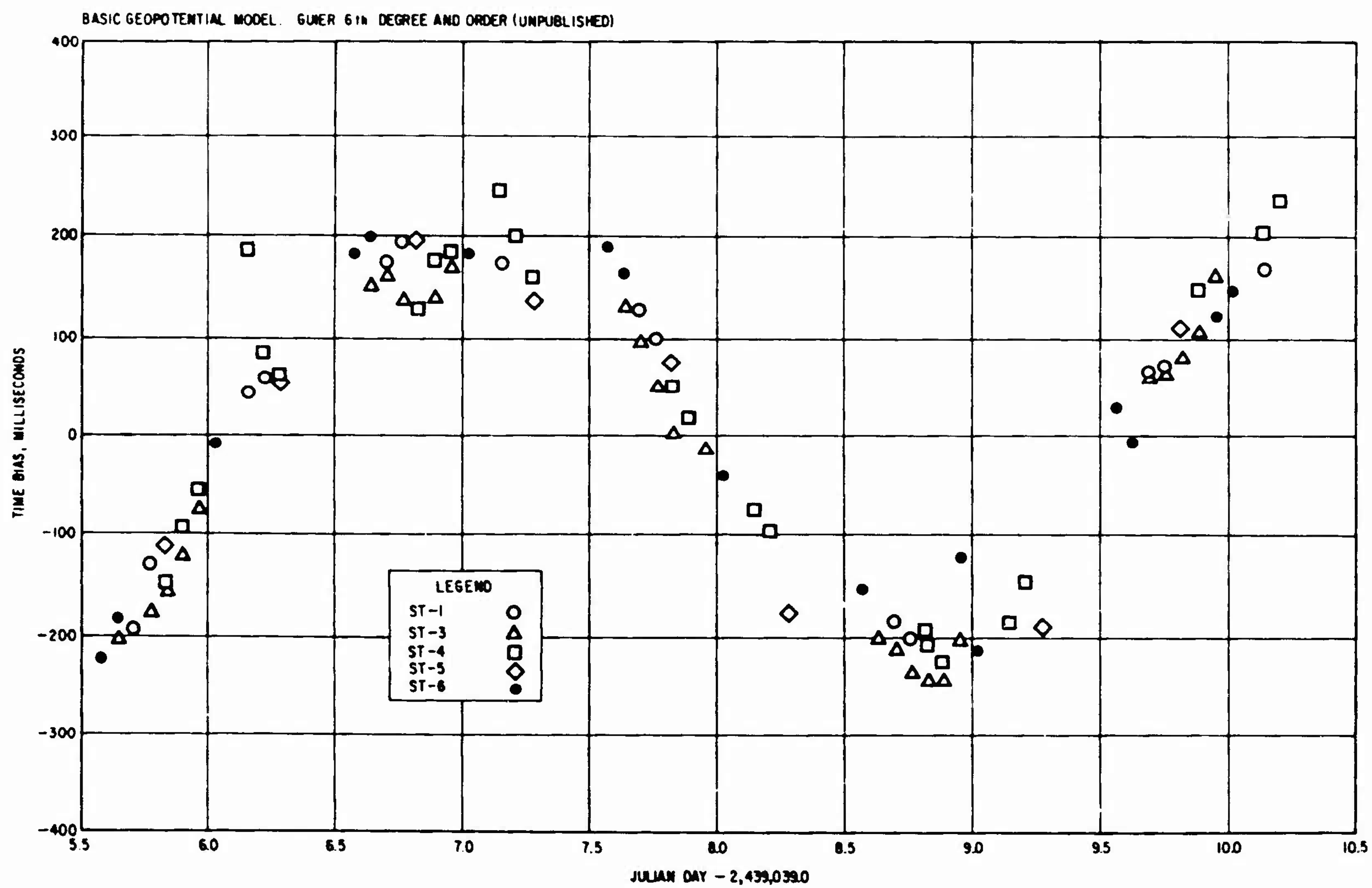

Figure 5. Time Bias Pattern without Resonant Terms: Satellite 1965 79A (Lockheed-Jacchia Atmospheric Density Model)

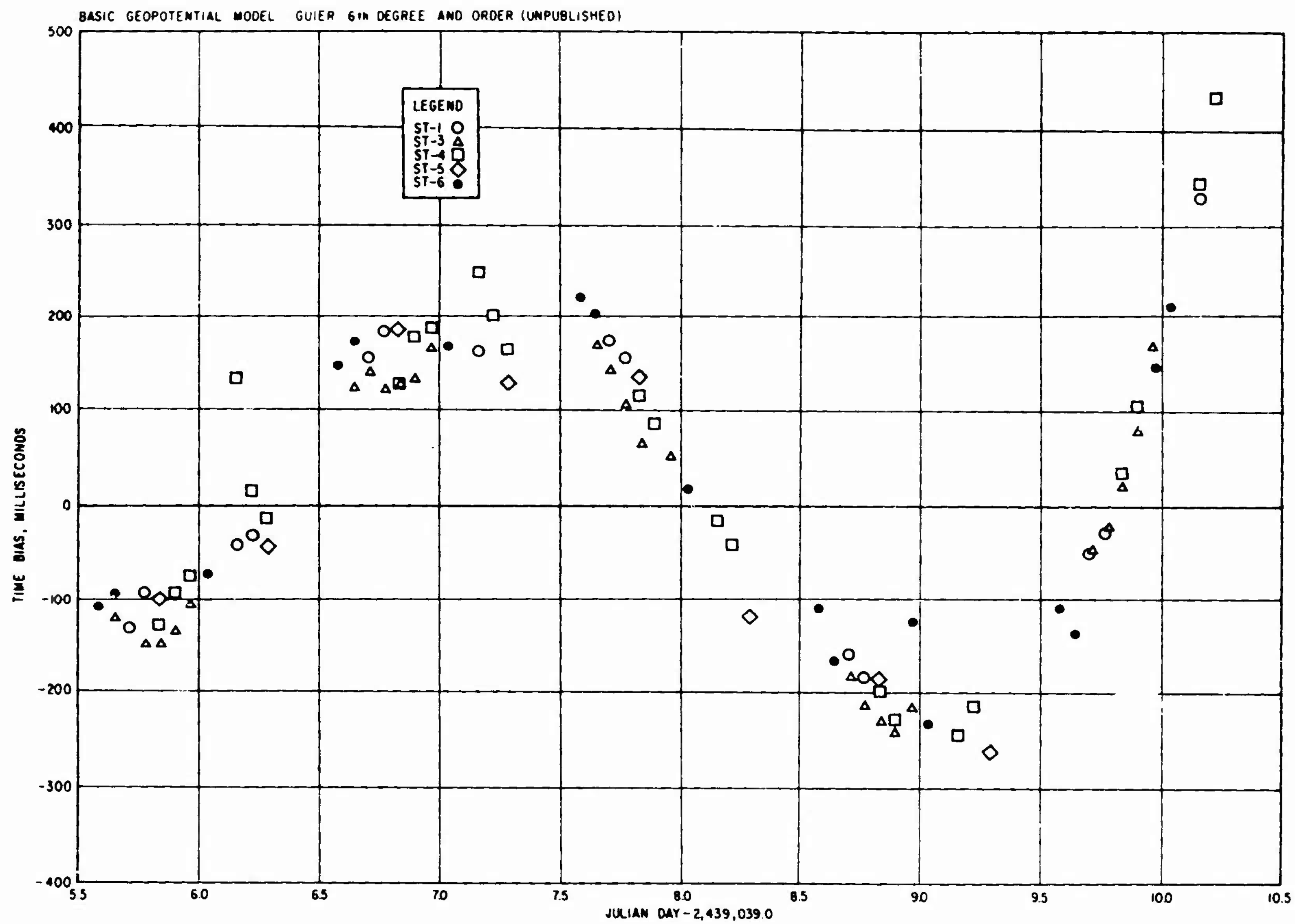

Figure 6. Time Bias Pattern without Resonant Terms: Satellite 1965 79A (Jacchia 1964 Atmospheric Density Model)

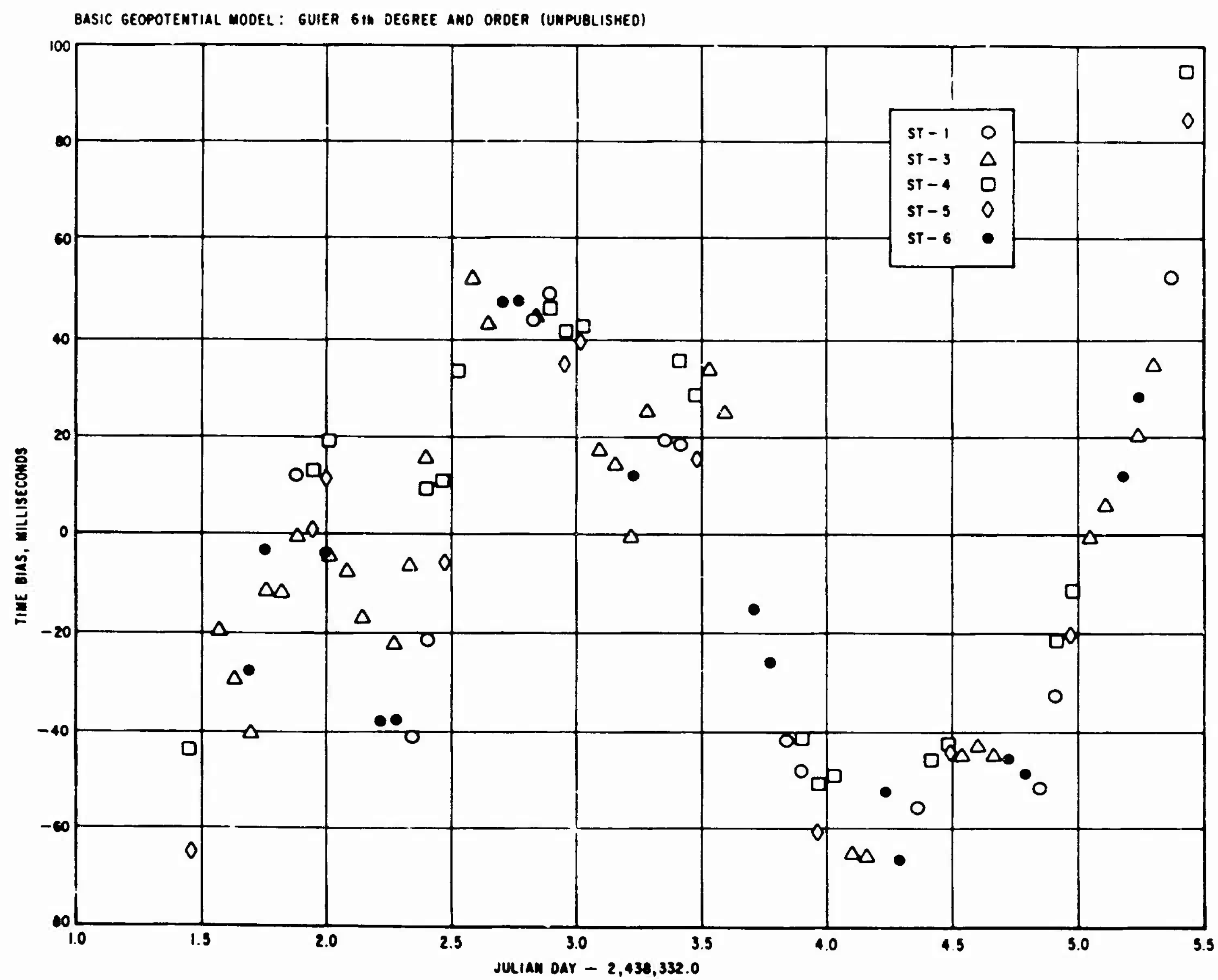

Figure 7. Time Bias Pattern without Resonant Terms: Satellite 1963 42A
(Lockheed-Jacchia Atmospheric Density Model)

is to show the extent to which these observed variations in the time bias pattern may be attributed to resonant tesseral terms omitted in the geopotential expansion. Having an observed value for the period of the resonance phenomenon from the time bias patterns, we next consider the theoretical prediction for likely geopotential terms.

From Reference 2, one may obtain the theoretically expected period of the resonance phenomenon P_R as

$$P_R = \frac{2\pi/\dot{U}}{|j + mp|} \qquad (2)$$

where m is the order of the geopotential term $j = 1, 2, 3, \cdots$, and

$$p = -\frac{\omega_e - \dot{\Omega}}{n + \dot{\omega}} \approx -\frac{1}{16} \qquad (3)$$

for the satellites discussed here.

In Eq. (3)

ω_e = the earth's sidereal rotation rate

n = Keplerian mean motion of the satellite

ω = argument of perigee

Ω = inertial longitude (right ascension) of the ascending node

The mean argument of latitude U is equal to tne sum of ω and the mean anomaly M.

As a very rough approximation, one may take

$$\dot{U} \approx n = \frac{2\pi}{P_K} \qquad (4)$$

and

$$p \approx -\frac{\omega_e}{n} = -\frac{P_K \omega_e}{2\pi} \qquad (5)$$

and generate curves such as those presented in Figure 8. One may then use Figure 8 to obtain a rather good qualitative estimate for nearness to resonance simply from the Keplerian period P_K.

A more accurate determination of P_R can be obtained by using the first-order oblateness expressions

$$\dot{M} = n\left[1 + \frac{3J_2}{4(1-e^2)^{3/2}}\left(\frac{a_e}{a}\right)^2 (3\cos^2 i - 1)\right] \tag{6}$$

$$\dot{\Omega} = -n\frac{3J_2}{2(1-e^2)^2}\left(\frac{a_e}{a}\right)^2 \cos i \tag{7}$$

and

$$\dot{\omega} = n\frac{3J_2}{4(1-e^2)^2}\left(\frac{a_e}{a}\right)^2 (5\cos^2 i - 1) \tag{8}$$

in Eqs. (2) and (3), where

J_2 = the coefficient of the second zonal harmonic
a = the major semi-axis
e = the eccentricity
i = the inclination of the satellite orbit.

Earlier a_e was defined. From the foregoing equations, one may construct tables of P_R for a given satellite as a function of j and m. Table II presents such data for satellites 1962 BE 1, 1963 55A, and 1965 79A.

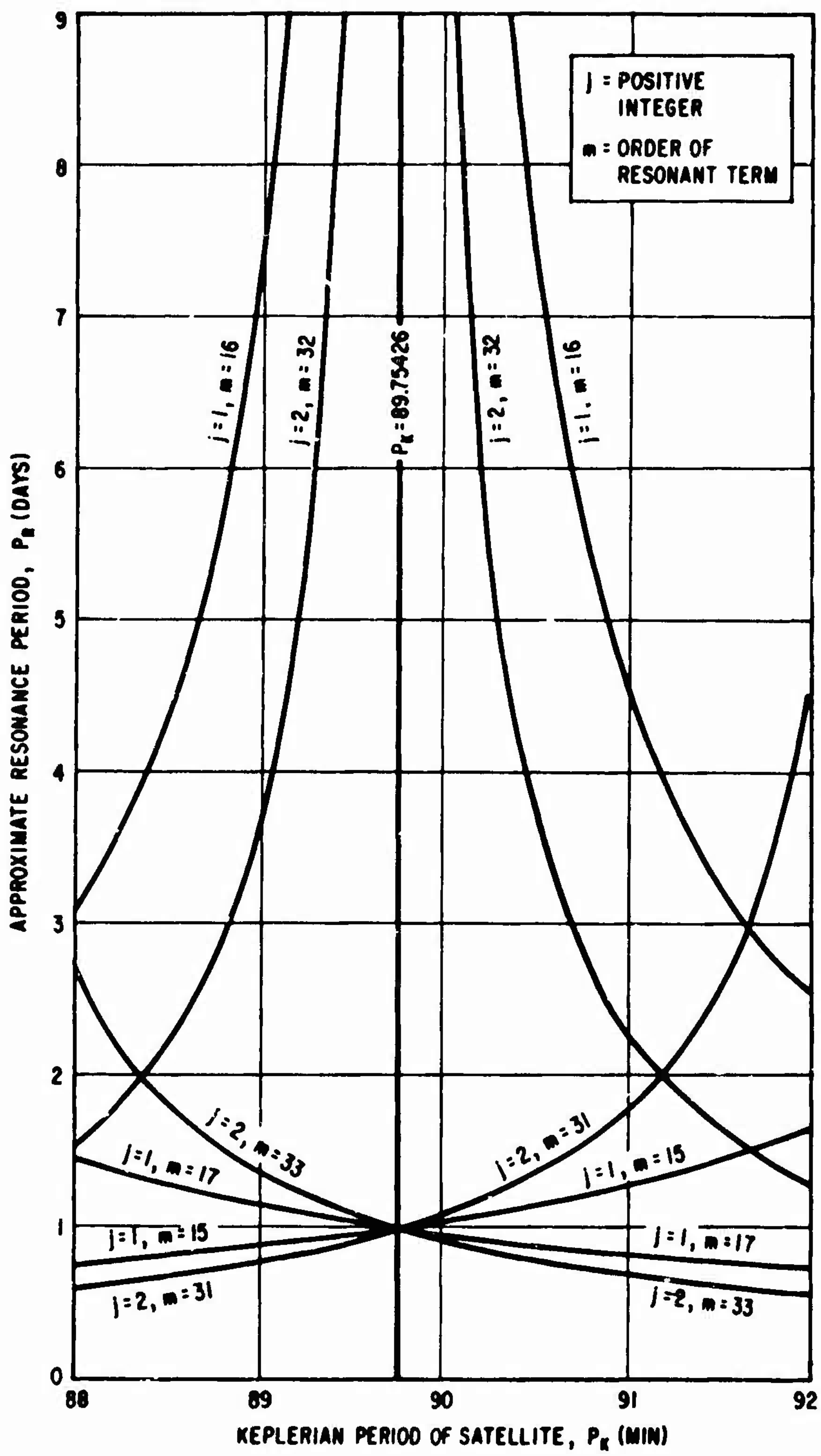

Figure 8. Approximate Resonance Period as a Function of Keplerian Period

Table II. Theoretical Values of the Period of Resonance P_R Phenomenon

Satellite	j	m	P_R (days)
1962 BE 1	1	15	1.4
	1	16	3.3
	1	17	0.8
	2	31	2.4
	2	32	1.7
	2	33	0.6
1963 55A	1	15	1.3
	1	16	4.5
	1	17	0.8
	2	31	1.8
	2	32	2.2
	2	33	0.7
1965 79A	1	15	1.1
	1	16	8.6
	1	17	0.9
	2	31	1.3
	2	32	4.3
	2	33	0.8

As an example of the use of Table II, a comparison of the 3.1-day period in the time bias patterns for satellite 1962 BE 1 (see Figures 1 and 2) with the entries in Table II strongly suggests that the missing resonant term would have order $m = 16$ (and $j = 1$). By returning to the theory presented in Reference 2, it can be seen that odd j signifies odd degree n, with $n = 17$ being the first odd degree for $m = 16$. Results for the tesseral term $(n, m) = (17, 16)$ on satellite 1962 BE 1 will be presented in the next section, along with those for the remainder of the flights in Table I. Table III presents a comparison of the observed and predicted resonance periods for satellites 1962 BE 1, 1963 55A, and 1965 79A, and the associated resonant term predicted. Similar small-percent discrepancies between the predicted and observed resonance periods noted in Table III were also reported in Reference 1.

Table III. Comparison of the Observed and Predicted Resonance Periods and Associated Resonance Term

Satellite	P_R Observed (days)	P_R Predicted (days)	m	n
1962 BE 1	3.1	3.3	16	odd
1963 55A	3.8	4.5	16	odd
1965 79A	3.5	4.3	32	even

SECTION III

RESULTS AND DISCUSSION

Table IV presents a summary of the results on resonance terms derived from the satellites listed in Table I. Figures 9 through 16 show the time bias patterns for satellites 1962 BE 1, 1963 55A, 1965 79A, and 1963 42A that prevailed when the derived values in Table IV were used in the force model. To determine how well the derived terms do remove the in-track error observed earlier, Figures 9 through 16 should be compared with the corresponding 7-vector solutions presented in Figures 1 through 7.

In all combinations of the satellites and atmospheric density models tested, one observes a nearly complete removal of all systematic variations in the time bias patterns of Figures 1 through 7 by the derived resonant term predicted for the individual satellites. Further, Table IV shows that the derived coefficients of the same degree and order are generally in close agreement with each other and with rule-of-thumb values suggested by Kaula.

Regarding this last statement, in a private communication Kaula has suggested that an examination of autocovariance analysis values indicates that $E\{\bar{J}_{nm}\} \approx 0.07 \times 10^{-6}$ for a degree n in the range 13 to 19. Unnormalizing this value for starting values for (n, m) = (17, 16) and (n, m) = (17, 17) yields, respectively, 2.0×10^{-25} and 3.4×10^{-26}, which are in generally close agreement with the values derived. This is particularly true of the higher-altitude satellites 1963 35A and 1963 42A, on which the effect of the atmosphere is less and there is less variation in results when changing atmospheric density models.

The discordant value obtained for (n, m) = (17, 16) from satellite 1962 BE 1 when using the Jacchia 1964 atmospheric density model has subsequently been shown to be due to the manner in which this model incorporates density variations correlated with the earth's magnetic activity. By setting the planetary magnetic amplitude at a constant value in the Jacchia 1964 model, one obtains $J_{17,16} = 3.29 \times 10^{-25}$ and $L_{17,16} = -13.58$ degrees from the data of satellite 1962 BE 1.

Table IV. Summary of Derived Resonance Terms

SATELLITE	n	m	J_{nm}	L_{nm}(deg)	ATMOSPHERE MODEL
1962 BE 1	17	16	4.29×10^{-25}	-13.45	Lockheed-Jacchia
	17	16	15.43×10^{-25}	-14.18	Jacchia 1964
	17	16	4.99×10^{-25}	-13.51	Lockheed-Jacchia
plus	32	32	1.80×10^{-51}	3.83	
1963 55A	17	16	9.32×10^{-25}	-9.84	Lockheed-Jacchia
	17	16	3.24×10^{-25}	-7.13	Jacchia 1964
1965 79A	32	32	4.29×10^{-51}	0.08	Lockheed-Jacchia
	32	32	6.28×10^{-51}	-1.33	Jacchia 1964
1963 35A	17	16	1.83×10^{-25}	-4.47	Lockheed-Jacchia
	17	16	1.80×10^{-25}	-13.34	Jacchia 1964
	17	16	2.62×10^{-25}	-8.50	Jacchia 1964
plus	17	17	2.50×10^{-26}	14.50	
plus	32	32	1.10×10^{-51}	6.86	
1963 42A	17	16	2.90×10^{-25}	-10.50	Lockheed-Jacchia
plus	17	17	3.70×10^{-26}	13.50	

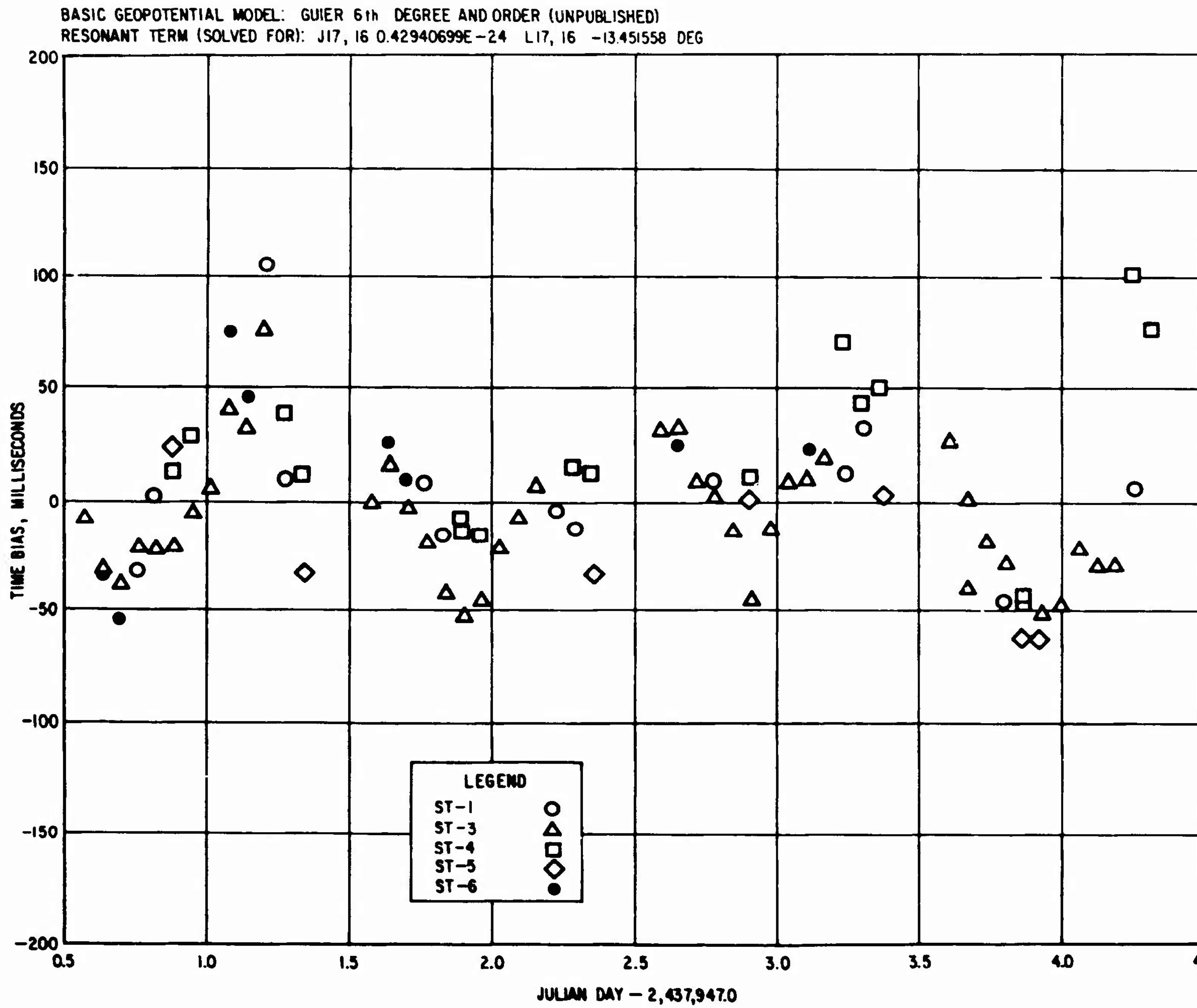

Figure 9. Time Bias Pattern with Resonant Terms: Satellite 1962 BE 1 (Lockheed-Jacchia Model)

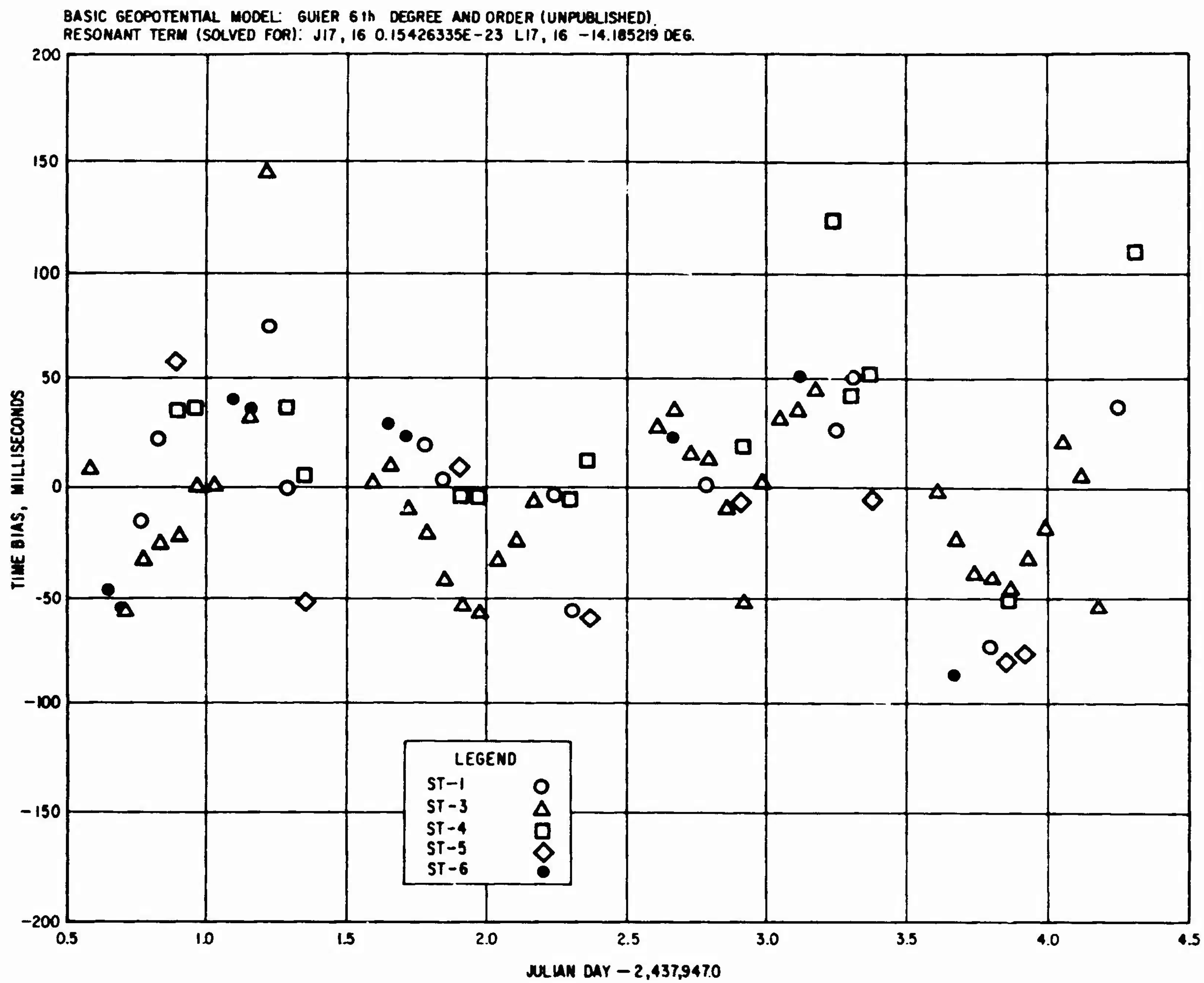

Figure 10. Time Bias Pattern with Resonant Terms: Satellite 1962 BE 1 (Jacchia 1964 Model)

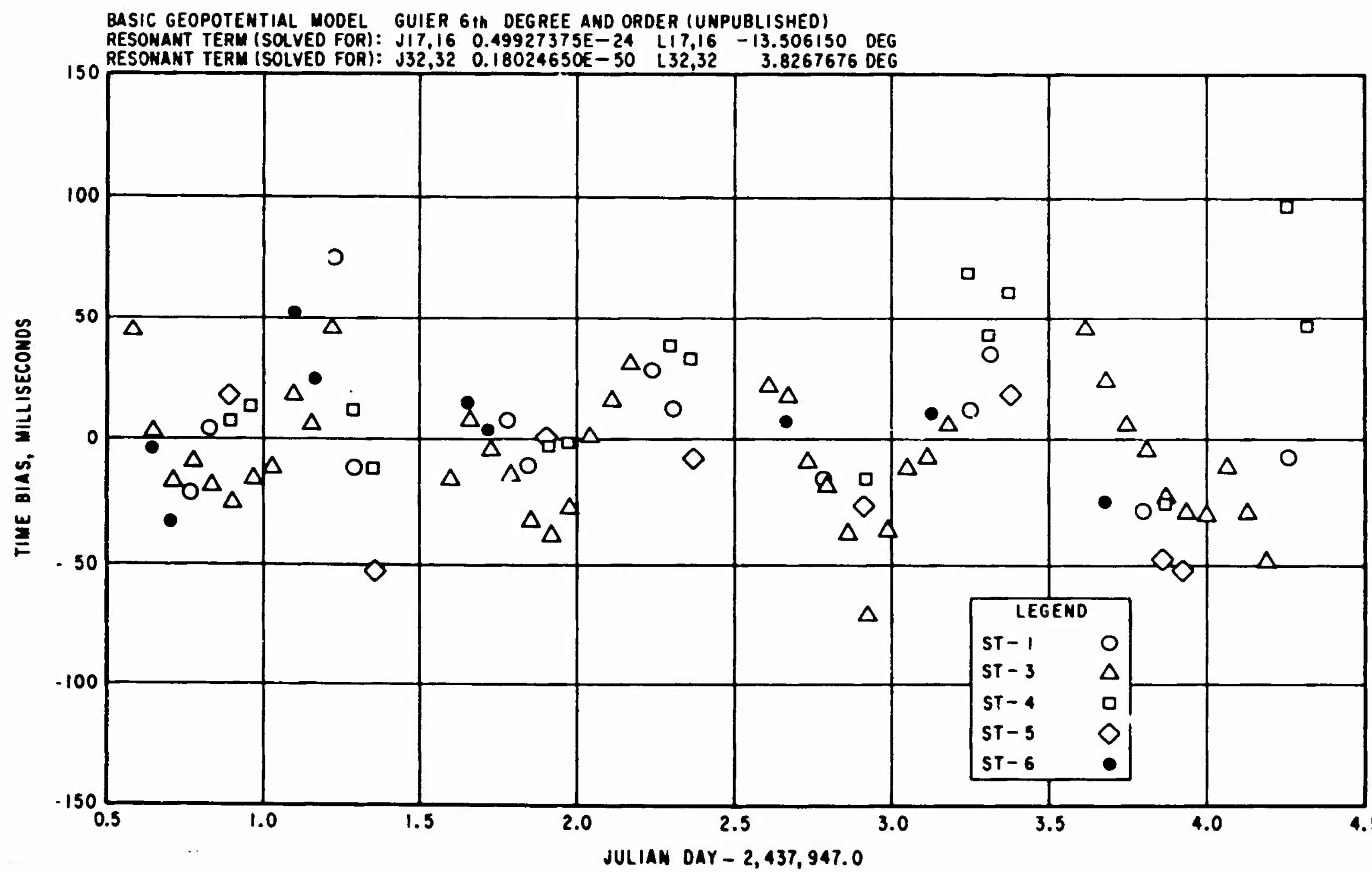

Figure 11. Time Bias Pattern with Resonant Terms: Satellite 1962 BE 1
(Lockheed-Jacchia Model)

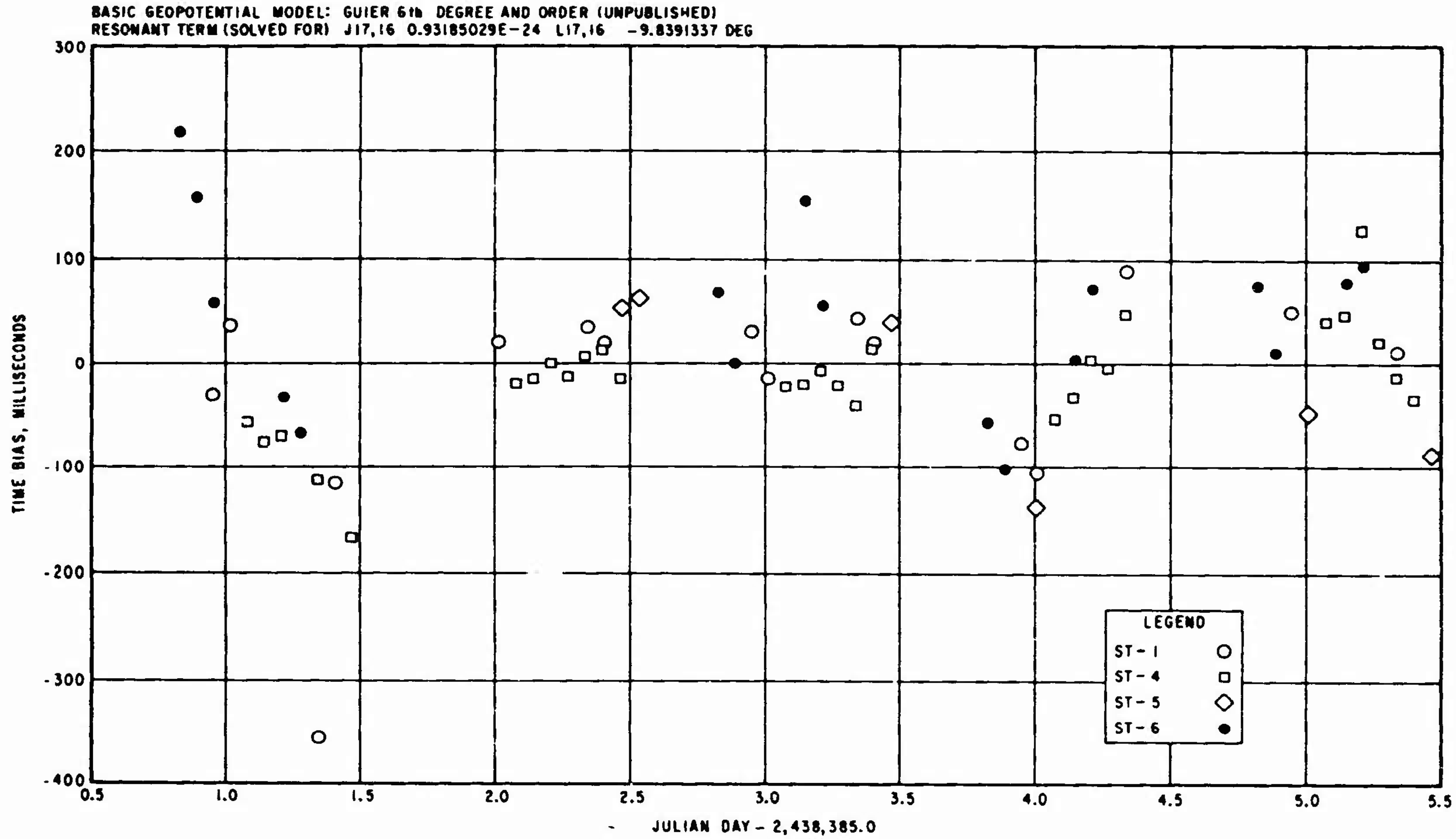

Figure 12. Time Bias Pattern with Resonant Terms: Satellite 1963 55A
(Lockheed-Jacchia Atmospheric Density Model)

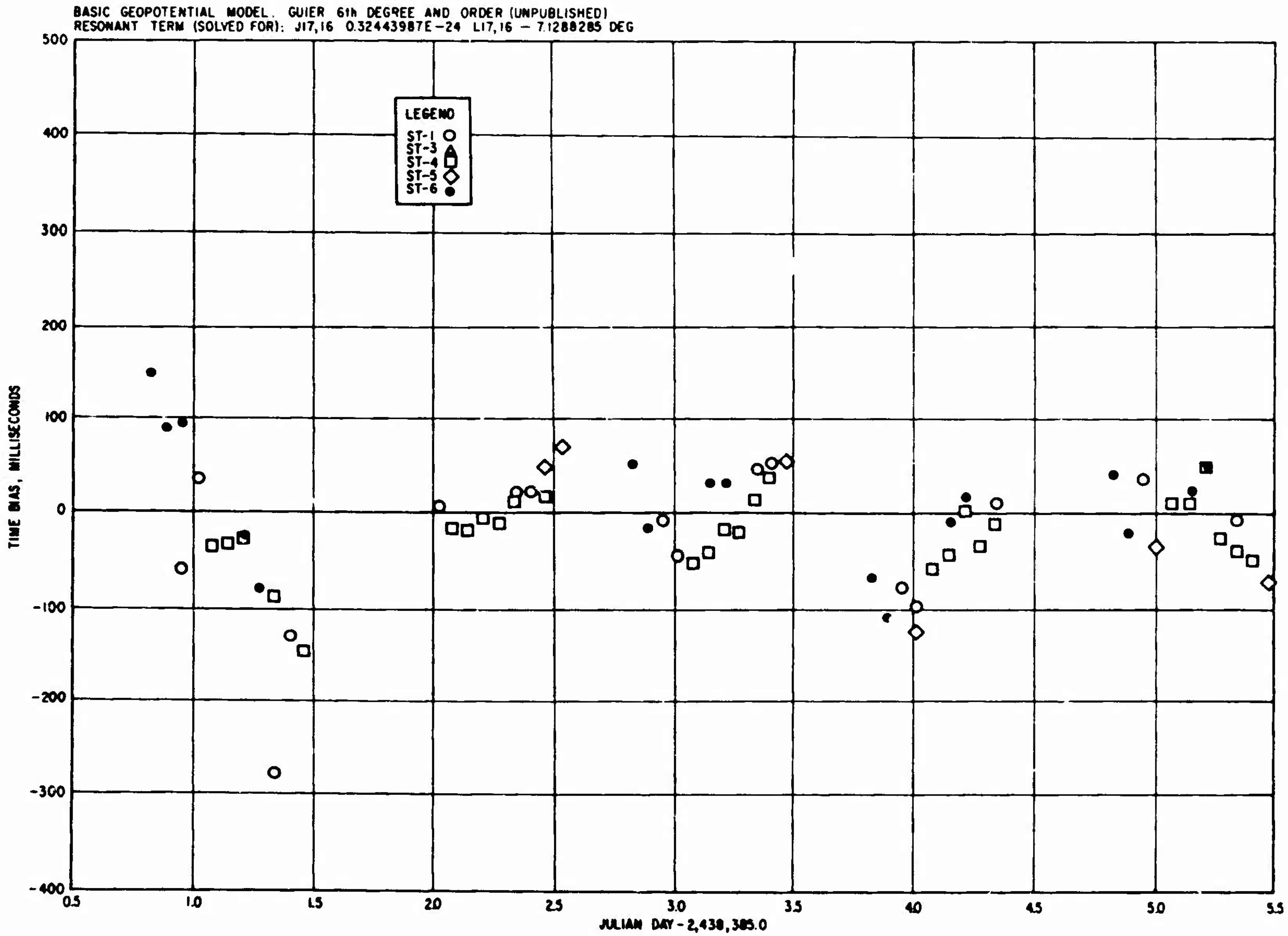

Figure 13. Time Bias Pattern with Resonant Terms: Satellite 1963 55A
(Jacchia 1964 Atmospheric Density Model)

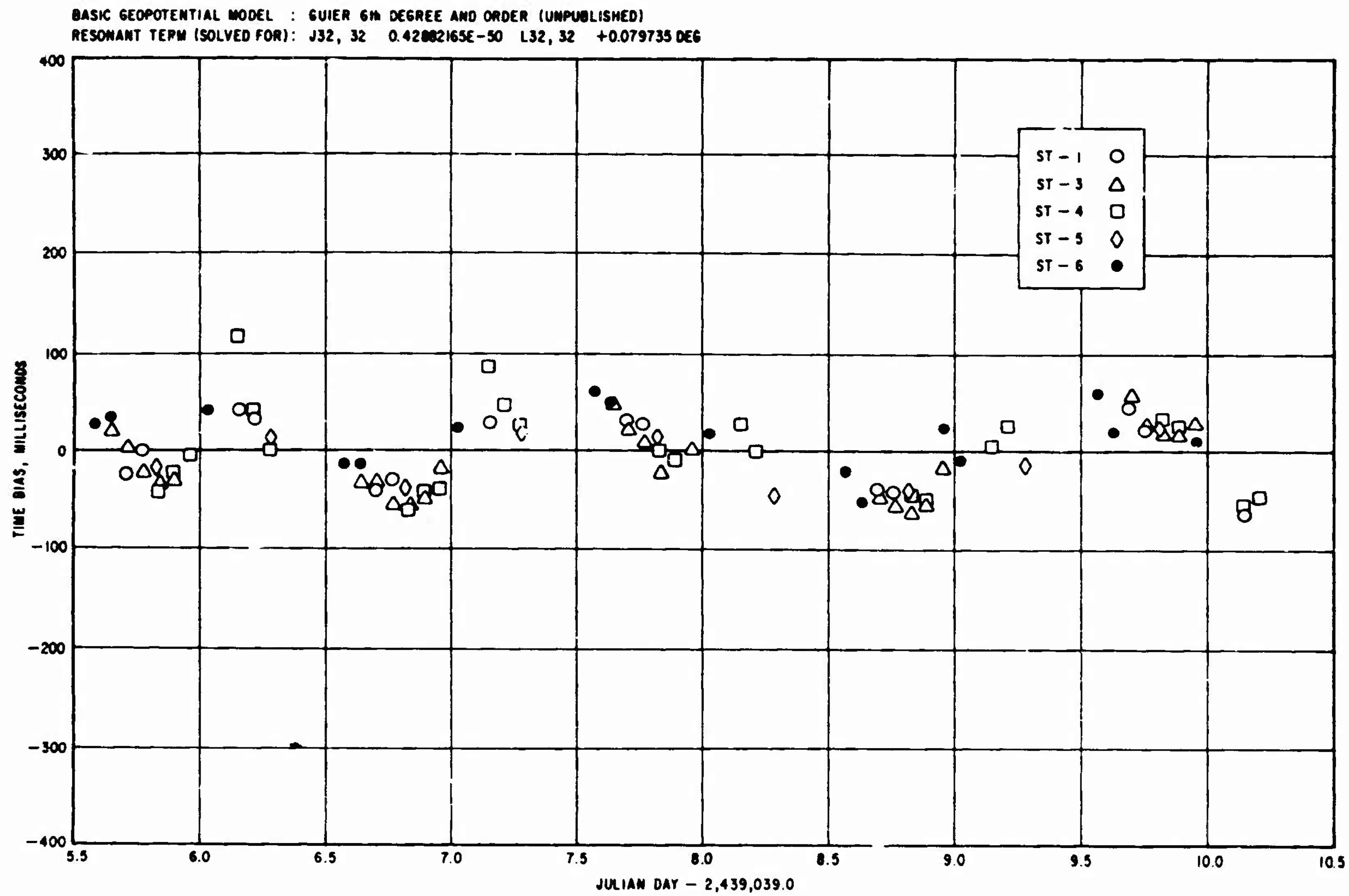

Figure 14. Time Bias Pattern with Resonant Terms: Satellite 1965 79A (Lockheed-Jacchia Atmospheric Density Model)

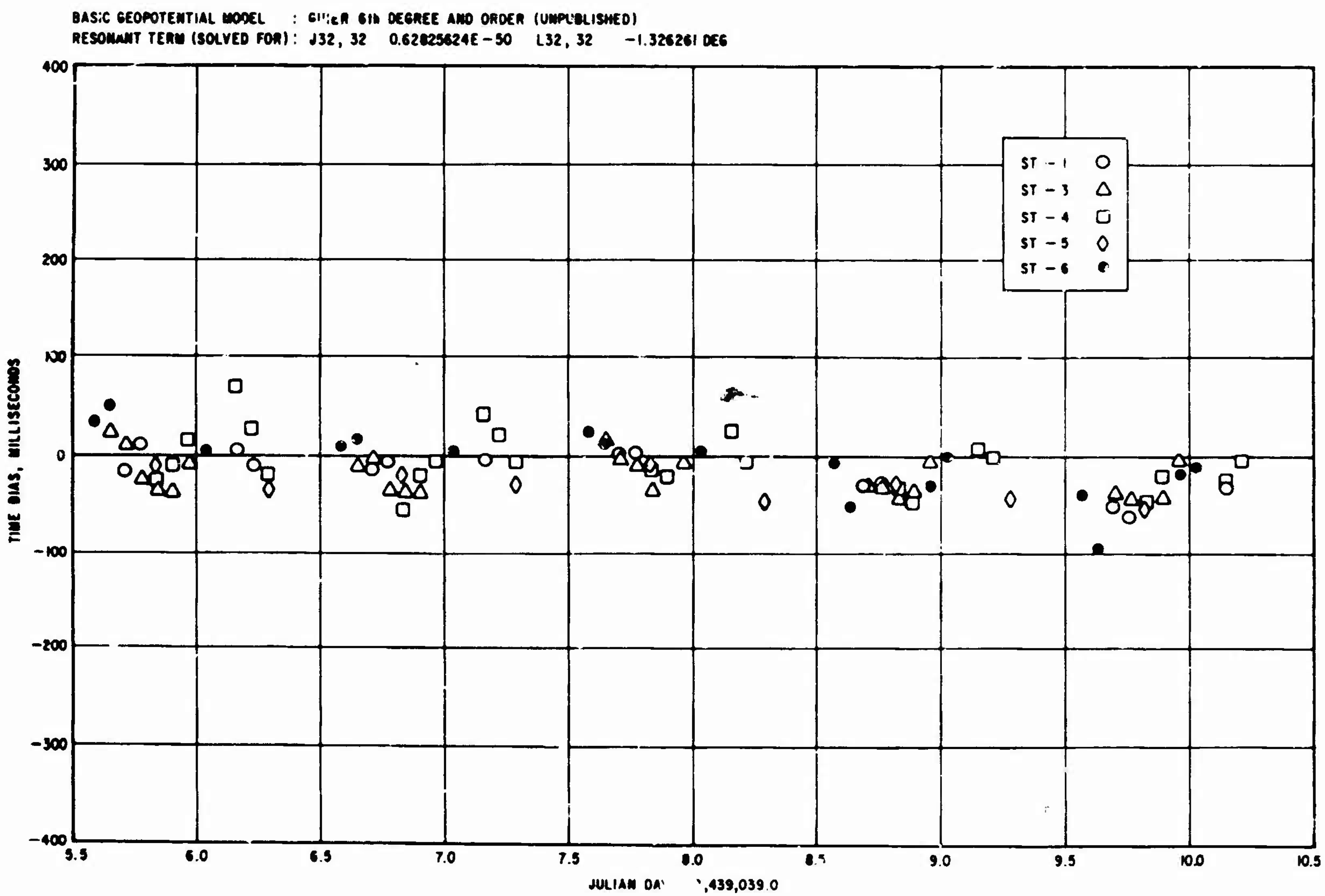

Figure 15. Time Bias Pattern with Resonant Terms: Satellite 1965 79A
(Jacchia 1964 Atmospheric Density Model)

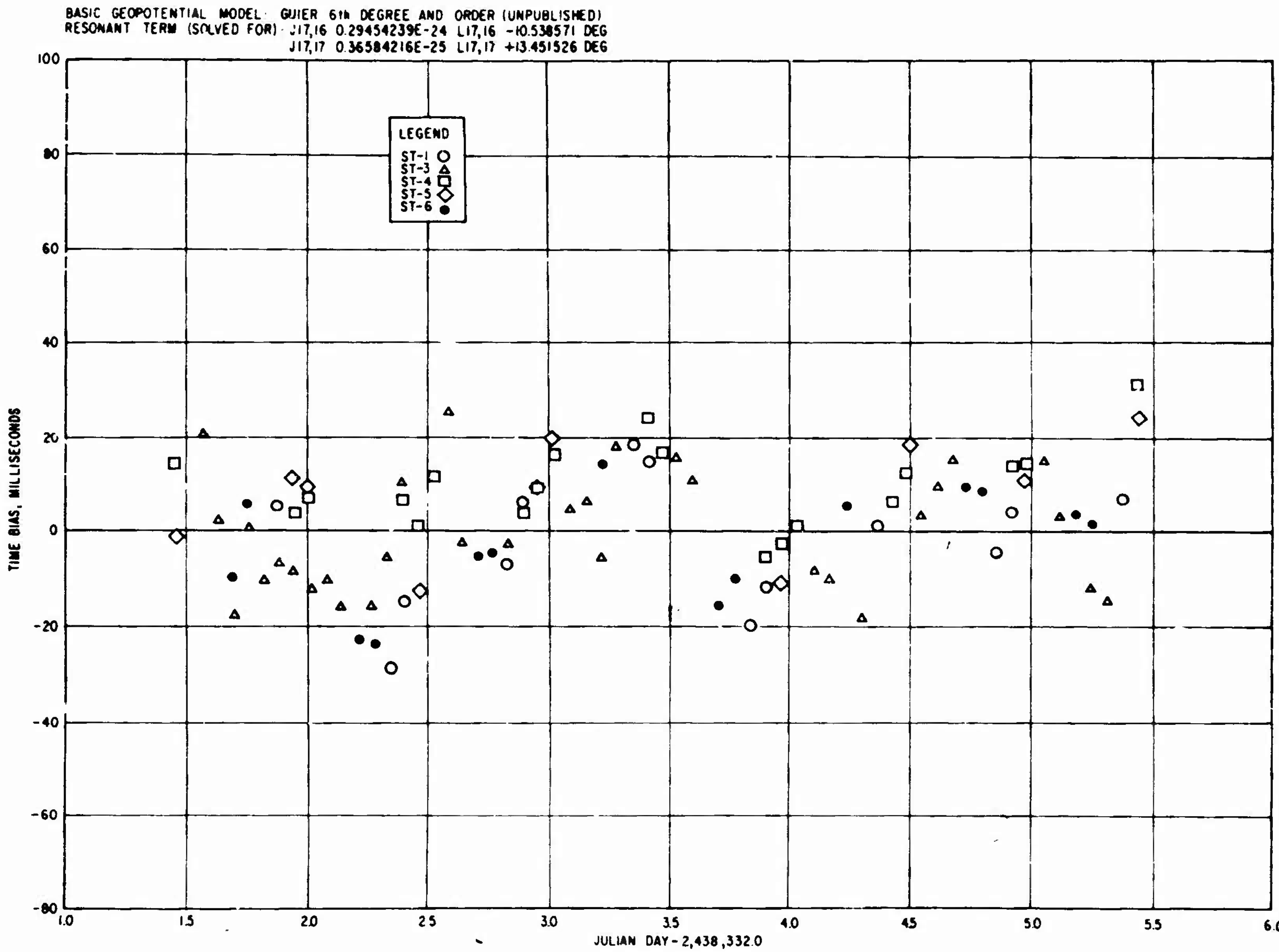

Figure 16. Time Bias Pattern with Resonant Terms: Satellite 1963 42A (Lockheed-Jacchia Atmospheric Density Model)

SECTION IV

CONCLUSION

Atmospheric density variations improperly modeled may contribute to the time bias patterns of Figures 1 through 7, but neither the Lockheed-Jacchia nor the Jacchia 1964 models represent perfectly the density variations experienced by these satellites. By recognizing these two facts simultaneously, it is concluded from the foregoing results that even in a high-drag environment one still observes an underlying satellite resonance phenomenon with geopotential forces arising from high-degree and order tesseral terms of the geopotential expansion.

It is recognized, however, that an accurate determination of the descriptive parameters would be extremely difficult from the present low-altitude satellite data, especially with the atmospheric density models presently available. Their determination must result from a joint geopotential-atmosphere study, which will refine both members of the force model. Otherwise the more exotic techniques of drag-free satellites (i. e., no drag force modeling required) or the direct use of drag accelerometer data must be awaited.

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13 ABSTRACT
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Abstract (Continued)

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